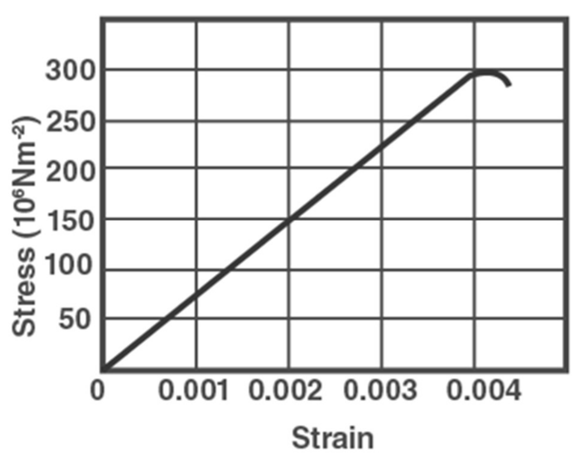
CH9 – MECHANICAL PROPERTIES OF SOLID

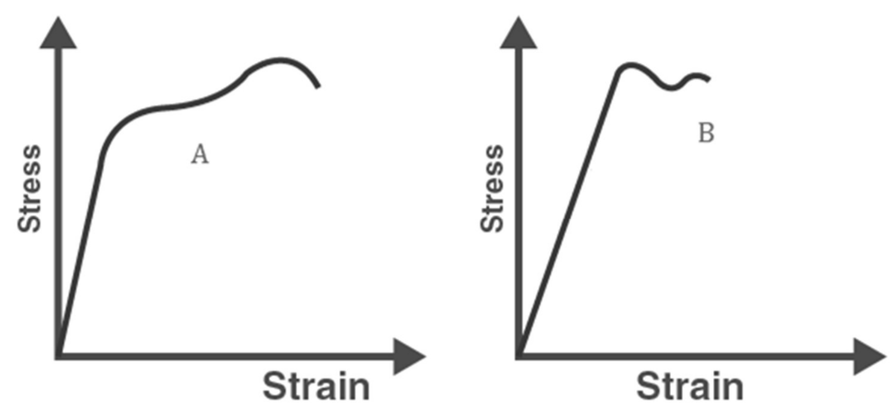
Q1. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper?
Answer:
Length of the steel wire, $l_1 = 4.7$ m
Cross-sectional area of the steel wire, $a_1 = 3.0 \times 10^{-5}$ m^2
Length of the copper wire, $l_2 = 3.5$ m
Cross-section area of the copper wire, $a_2 = 4.0 \times 10^{-5}$ m^2
Change in length $= \Delta l_1 = \Delta l_2 = \Delta l$
Force applied in both cases = F
Young's modulus of the steel wire

——(1)
Young's modulus of the copper wire

——–(2)
Dividing (1) by (2), we get
Q2. The figure below shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



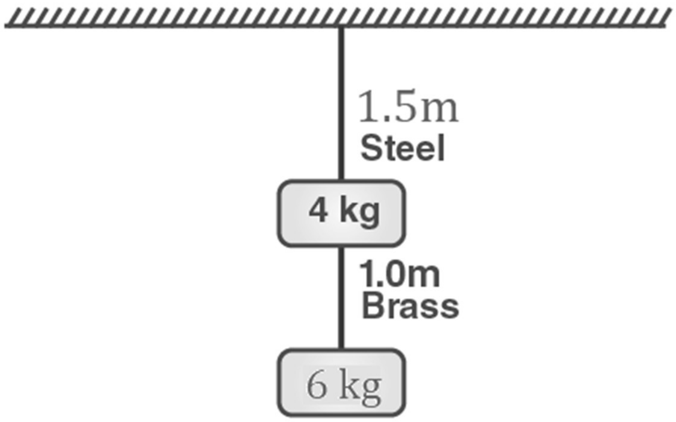
Answer:
Young's modulus ,Y =Stress/Strain
$=150 \times 10^6/0.002$
$= 150 \times 10^6/2 \times 10^{-3}$
$= 75 \times 10^9$ Nm^{-2}
$=75 \times 10^{10}$ Nm^{-2}
(a)Yield strength of a material is the maximum stress that the material can sustain and retain its elastic property. From the graph, the approximate yield strength of the given material
$= 300 \times 10^6$ Nm^{-2}
$= 3 \times 10^8$ Nm^{-2} .
Q3. The stress-strain graphs for materials A and B are shown in the figure below.



The graphs are drawn to the same scale.
(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?
Answer:
Young's modulus = Stress/Strain
(a) From the graphs, we can see that for the given strain, stress for A is greater than that of B. Therefore, Young's modulus of A is greater than B.
(b) Young's modulus is also a measure of the strength of the material. Young's modulus is greater for A; therefore, material A is stronger than B.
Q4. Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.
Answer.

(a) True. Stretching a coil does not change its length; only its shape is altered, and this involves shear modulus.
(b) False. This is because, for the same value of stress, there is more strain in rubber than in steel. And as Young Modulus is an inverse of strain, it is greater in steel.

Q5. Two wires of diameter 0.25 cm, one made of steel and the other made of brass, are loaded, as shown in Fig. The unloaded length of steel wire is 1.5 m, and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires. [Young's modulus of steel is 2.0 x 10^{11} Pa. (1 Pa = 1 N m^2)]



Answer:
Diameter of the two wires, d =0.25m
Radius of the wires, r= d/2 =0.125cm
Unloaded length of the steel wire, l_1 =1.5m
Unloaded length of the brass wire, l_2 =1.0m
Force exerted on the steel wire
$F_1 = (4+6)g = 10 \times 9.8 = 98N$
Cross-section area of the steel wire, $a_1 = \pi r_1^2$
Change in length of the steel wire = Δl_1
Young's modulus for steel= 2.0 x 10^{11} Pa

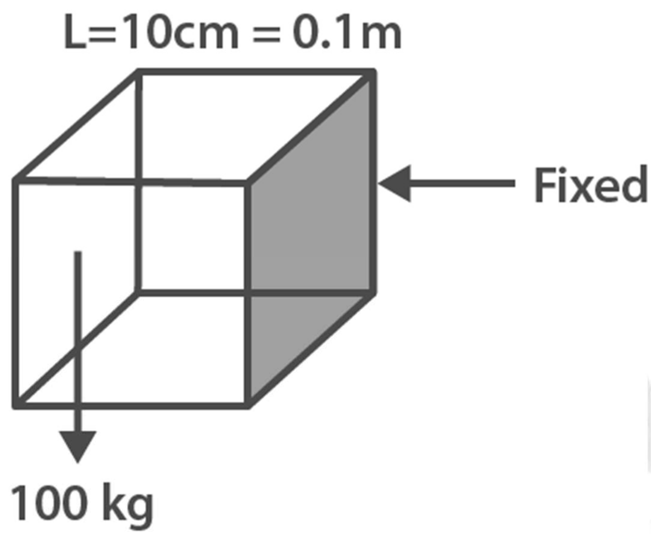
= 1.49 x 10^{-4} m
Force of the brass wire ,F_2 = 6 x 9.8 = 58. 8 N
Cross-section area of the brass wire, $a_2 = \pi r_2^2$

Change in length of the brass wire = Δl_2
Young's modulus of the brass wire = 0.91×10^{11} Pa

= 1.3×10^{-4} m
The elongation of the steel wire is 1.49×10^{-4} m, and that of brass is 1.3×10^{-4} m.

Q6. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Answer:



Edge of the aluminium cube, L = 10 cm =10/100= 0.1 m
Area of each face, A = $(0.1)^2 = 0.01$ m^2
Mass attached to the opposite face of the cube = 100 kg
Tangential force acting on the face, F = 100 kg = 100 x 9.8 = 980 N
Shear modulus, η = Tangential stress/Shearing strain
Shearing strain = Tangential stress/ Shear modulus
= F/Aη = $980/(0.01 \times 25 \times 10^9) = 3.92 \times 10^{-6}$
Shearing strain = Lateral strain/Side of the cube
Lateral strain = Shearing strain x Side of the cube = $3.92 \times 10^{-6} \times 0.1$
= 3.92×10^{-7} m ≈ 4×10^{-7} m

Q7. Four identical hollow cylindrical columns of mild steel support a big structure with a mass of 50,000 kg. The inner and outer radii of each

column are 30 and 60 cm, respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

Answer:

Mass of the big structure, M = 50,000 kg

Total force exerted on the four columns= Total weight of the structure=50000×9.8N

The compressional force on each column = Mg/4 = (50000×9.8)/4 N= 122500 N

Therefore, Stress = 122500 N

Young's modulus of steel, $Y=2\times10^{11}$ Pa

Young's modulus, Y= Stress/Strain

Strain = Young's modulus/Stress

Strain = (F/A)/Y

Inner radius of the column, r = 30 cm = 0.3 m

Outer radius of the column, R = 60 cm = 0.6 m

Where,

Area, $A=\pi(R^2-r^2)=\pi((0.6)^2-(0.3)^2) = 0.27\ \pi\ m^2$

Strain $=122500/[0.27 \times 3.14\times2\times10^{11}]=7.22\times10^{-7}$

Hence, the compressional strain of each column is 7.22×10^{-7}.

Q8. A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.

Answer:

Area of the copper piece, $A=19.1\times10^{-3}\times15.2\times10^{-3}=2.9\times10^{-4}m^2$

Tension force applied on the piece of copper, F=44,500 N

Modulus of elasticity of copper, $Y=42\times10^9\ Nm^{-2}$

Modulus of elasticity (Y) = Stress / Strain

=(F/A) / Strain

Strain = F/(YA)

$= 44500/(2.9\times10^{-4}\times42\times10^9)$

$= 3.65\times10^{-3}$

Q9. A steel cable with a radius of 1.5 cm supports a chairlift in a ski area. If the maximum stress is not to exceed $10^8\ Nm^{-2}$.What is the maximum load the cable can support?

Answer:

Radius of the steel cable, r=1.5 cm = 0.015 m

Cross-sectional area of the cable = $\pi r^2= 3.14 \times (0.015)^2$

$= 7.06 \times 10^{-4}$ m
Maximum stress allowed on the steel cable = 10^8 N/m^2
Maximum load the cable can support = Maximum stress × Area of cross-section
$= 10^8 \times 7.06 \times 10^{-4}$
$= 7.065\times10^4$ N
Hence, the cable can support the maximum load of 7.065×10^4 N.

Q10. A rigid bar of mass 15 kg is supported symmetrically by three wires, each 2.0 m long. Those at each end are of copper, and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Answer.
As the tension on the wires is the same, the extension of each wire will also be the same. Now, as the length of the wires is the same, the strain on them will also be equal.
Now, we know
Y = Stress / Strain
= (F/A) / Strain = $(4F/\pi d^2)$ / Strain .... (1)
Where,
A = Area of cross-section
F = Tension force
d = Diameter of the wire
We can conclude from equation (1) that $Y \propto (1/d^2)$.
We know that Young's modulus for iron, $Y_1 = 190 \times 10^9$ Pa
Let the diameter of the iron wire = d_1
Also, Young's modulus for copper, $Y_2 = 120 \times 10^9$ Pa
Let the diameter of the copper wire = d_2
Thus, the ratio of their diameters can be given as
=
=1 : 25 : 1

Q11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.
Answer:
Mass, m = 14.5 kg

Length of the steel wire, l = 1 m
Angular velocity, v = 2 rev/s
Cross-sectional area of the wire, A = 0.065×10^{-4} m^2
Total pulling force on the steel wire when the mass is at the lowest point of the vertical circle, $F = mg + mr\omega^2$
$= 14.5\times9.8+14.5\times1\times(12.56)^2$
=2429.53 N
Young's modulus = Stress / Strain

$\Delta l=(2429.53\times1)/(0.065\times10^{-4}) \times (2\times10^{11}) = 1.87\times10^{-3}$m
Hence, the elongation of the wire when the mass is at the lowest is 1.87×10^{-3}m.

Q12. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litres, Pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), Final volume = 100.5 litres. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Answer:
Initial volume, $V_1 = 100.0$ litre $=100.0\times10^{-3}$ m^3
Final volume, $V_2 = 100.5$ litre $=100.5\times10^{-3}$ m^3
Change in the volume, $\Delta V=V_2 - V_1 = 0.5\times10^{-3}$ m^3
Pressure increase, $p=100.0\text{atm}=100\times1.013\times10^5$Pa
$= 101.3 \times 10^5$Pa
Bulk modulus of water= $p/(\Delta V/V_1)=pV_1/\Delta V$
$= 101.3\times10^5\times100\times10^{-3}/(0.5\times10^{-3})$
$= 2.026\times10^9$ Pa
Bulk modulus of air = 1×10^5 Pa
Bulk modulus of water / Bulk modulus of air = $2.026\times10^9/(1\times10^5)$
$=2.026\times10^4$
The intermolecular force in liquids is much larger than in air as the distance between the molecules is much lesser in liquid than in air. Therefore, at the same temperature, the strain for water is much more than for air.

Q13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3}?

Answer.
Let the depth be the alphabet 'd'.

Given,
Pressure at the given depth, p = 60.0 atm = $60 \times 1.01 \times 10^5$ Pa
Density of water at the surface, $\rho_1 = 1.03 \times 10^3$ kg m^{-3}
Let ρ_2 be the density of water at the depth d.
V_1 be the volume of water of mass m at the surface.
Then, let V_2 be the volume of water of mass m at the depth h.
And ΔV is the change in volume.
$\Delta V = V_1 - V_2$
$= m [(1/\rho_1) - (1/\rho_2)]$ ∴ Volumetric strain = $\Delta V / V_1$
$= m [(1/\rho_1) - (1/\rho_2)] \times (\rho_1 / m)$
$\Delta V / V_1 = 1 - (\rho_1/\rho_2)$ . . . . (1)
We know, Bulk modulus, $B = pV_1 / \Delta V$
=> $\Delta V / V_1 = p / B$
Compressibility of water = (1/B) = 45.8×10^{-11} Pa^{-1}
∴ $\Delta V / V_1 = 60 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 2.78 \times 10^{-3}$ . . . . . (2)
Using equation (1) and equation (2), we get
$1 - (\rho_1/\rho_2) = 2.78 \times 10^{-3}$
$\rho_2 = 1.03 \times 10^3 / [1 - (2.78 \times 10^{-3})] = 1.032 \times 10^3$ kg m^{-3}
Therefore, at depth d, water has a density of 1.034×10^3 kg m^{-3}.

Q14. Compute the fractional change in volume of a glass slab when subjected to a hydraulic pressure of 10 atm.

Answer.
Given,
Pressure acting on the glass plate, p = 10 atm = $10 \times 1.013 \times 10^5$ Pa
We know,
Bulk modulus of glass, $B = 37 \times 10^9$ Nm^{-2}
=> Bulk modulus, $B = p / (\Delta V/V)$
Where,
$\Delta V/V$ = Fractional change in volume
∴ $\Delta V/V = p / B$
$= [10 \times 1.013 \times 10^5] / (37 \times 10^9)$
$= 2.73 \times 10^{-4}$
Therefore, the fractional change in the volume of the glass plate is 2.73×10^{-4}.

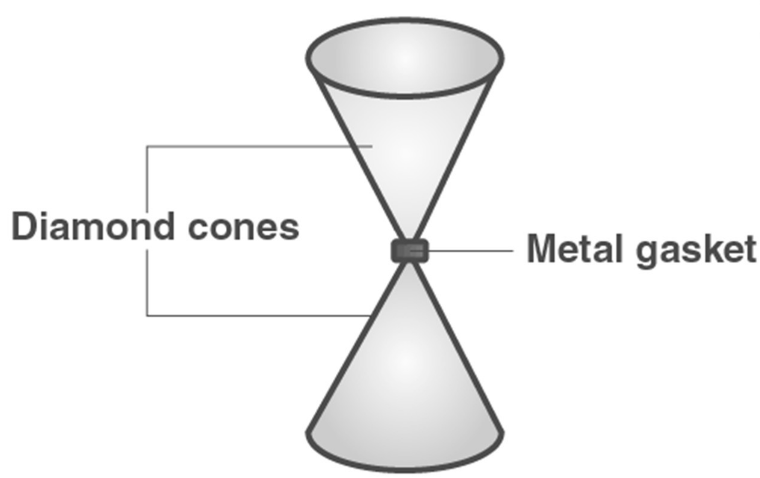
Q15. Determine the volume contraction of a solid copper cube, 10 cm on edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.
Answer:

Side of the copper cube, a = 10 cm
Therefore, the volume of the copper cube, $V = a^3 = 10^{-3}\ m^3$
Hydraulic pressure, $p = 7.0 \times 10^6$ Pa
Bulk modulus of copper $B = 140$ G Pa $= 140 \times 10^9$ Pa.
Bulk modulus, $K = -P/(\Delta V/V)$
We get the value of volume contraction as $\Delta V = -PV/K$
$= -(7\times10^6 \times 0.001)/(140\times10^9)$
$= -0.05\times10^{-6} m^3$

Q16.How much should the pressure on a litre of water be changed to compress it by 0.10%?

Answer:
Volume of water, V=1 litre
Water should be compressed by 0.10%
The fractional change in volume, $\triangle V/V = (0.1/100)\times1 = 10^{-3}$
Bulk modulus, $B = P/(\triangle V/V) = PV/\triangle V$
$P = B\times(\triangle V/V)$
Bulk modulus of water, $B = 2.2\times10^9\ Nm^{-2}$
Pressure on water, $P = 2.2\times10^9 \times10^{-3} = 2.2\times10^6$ Pa

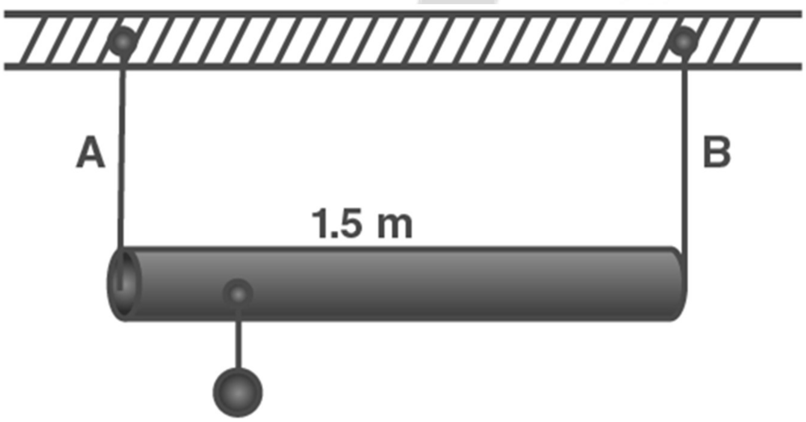
Q17. Anvils made of single crystals of diamond, with the shape as shown in the figure, are used to investigate the behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Answer:

Flat faces at the narrow end of the anvil have a diameter,
$d=0.50mm=0.5\times10^{-3}$ m
Radius, $r=d/2=0.25\times10^{-3}$ m
Compressional force, F=50000N
Pressure at the tip of the anvil
P=Force/Area
Area = $\pi r^2 = 3.14 \times (0.25\times10^{-3})^2 = 0.1925 \times 10^{-6} m^2$
Pressure at the tip of the anvil = F/A
= $50000/0.1925 \times 10^{-6}$
=2.59×10^{11} Pa

Q18. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths, as shown in the figure. The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm², respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires?



Answer.
Given,
Cross-sectional area of wire A, $a_1 = 1.0\ mm^2 = 1.0 \times 10^{-6}\ m^2$
Cross-sectional area of wire B, $a_2 = 2\ mm^2 = 2 \times 10^{-6}\ m^2$
We know Young's modulus for steel, $Y_1 = 2 \times 10^{11}\ Nm^{-2}$
Young's modulus for aluminium, $Y_2 = 7.0 \times 10^{10}\ Nm^{-2}$

(i) Let a mass m be hung on the stick at a distance y from the end where wire A is attached.
Stress in the wire = Force / Area = F / a

Now, it is given that the two wires have equal stresses

$F_1 / a_1 = F_2 / a_2$

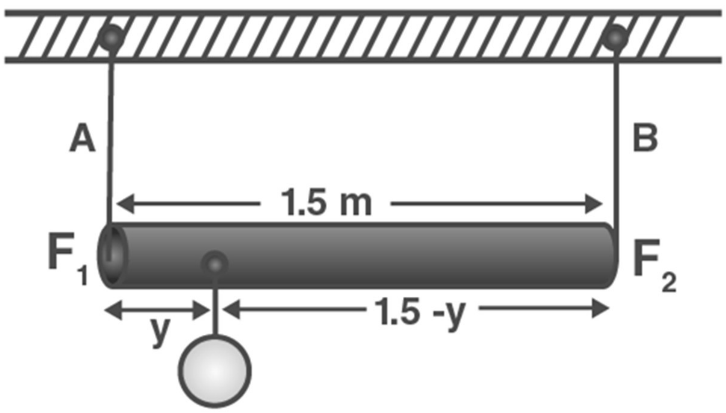
Where,

F_1 = Force acting on wire A

and F_2 = Force acting on wire B

$F_1 / F_2 = a_1 / a_2 = 1 / 2$ .... (1)

The above situation can be represented as



Moment of forces about the point of suspension, we have

$F_1 y = F_2 (1.5 – y)$

$F_1 / F_2 = (1.5 – y) / y$ .... (2)

Using equation (1) and equation (2), we can write

$(1.5 – y) / y = 1 / 2$

$2 (1.5 – y) = y$

$y = 1$ m

Therefore, the mass needs to be hung at a distance of 1m from the end where wire A is attached in order to produce equal stress in the two wires.

(ii) We know,

Young's modulus = Stress / Strain

=> Strain = Stress / Young's modulus = (F/a)/ Y

It is given that the strain in the two wires is equal

$(F_1/a_1) / Y_1 = (F_2/a_2) / Y_2$

$F_1 / F_2 = a_1Y_1 / a_2Y_2$

$a_1 / a_2 = 1 / 2$

$F_1 / F_2 = (1 / 2) (2 \times 10^{11} / 7 \times 10^{10}) = 10 / 7$ .... (3)

Let the mass m be hung on the stick at a distance y_1 from the end where the steel wire is attached in order to produce equal strain.

Taking the moment of the force about the point where mass m is suspended

$F_1y_1 = F_2 (1.5 – y_1)$

$F_1 / F_2 = (1.5 – y_1) / y_1$ ....(4)

From equations (3) and (4), we get

$(1.05 – y_1) / y_1 = 10 / 7$

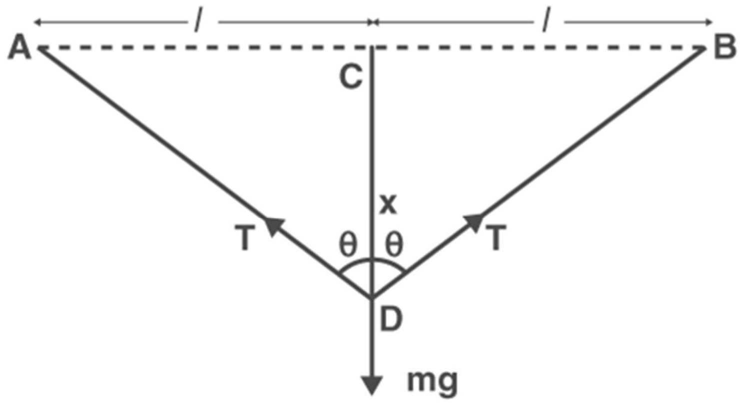
$7(1.05 – y_1) = 10y_1$

$y_1 = 0.432$ m

Therefore, the mass needs to be hung at a distance of 0.432 m from the end where wire A is attached in order to produce equal strain in the two wires.

Q19. 9 A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm^2 is stretched well within its elastic limit horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

Answer.



Steel wire length = 1.0 m

Area of cross-section, A = 0.50

Mass of the given object, m = 100 g = 0.1 kg

From the given diagram,

We can deduce that

is the depression at the midpoint, that is,

CD =

It is given that,

AD = BD =

AC = CB = I = 0.5m

m = 100g = 0.100kg

Gain in length,

= AD + DB – AB = 2AD – AB

Thus, strain =

If the tension of the wire is
, then
∴ Now,
Since,
<< 1, then 1 >>
&
Therefore,
Stress =
Young's modulus
Therefore,

Q20. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one-quarter of the load.

Answer:
Diameter of the metal strips= 6mm = 6×10^{-3} m
Radius, $r = 3 \times 10^{-3}$ m;
Shearing stress on the rivet= 6.9×10^7 Pa
Maximum load or force on a rivet
= Maximum stress x cross-sectional area
= $6.9 \times 10^7 \times 3.14 \times (3 \times 10^{-3})^2$ N = 1950 N
Maximum tension = 4 x 1950 N = 7800 N

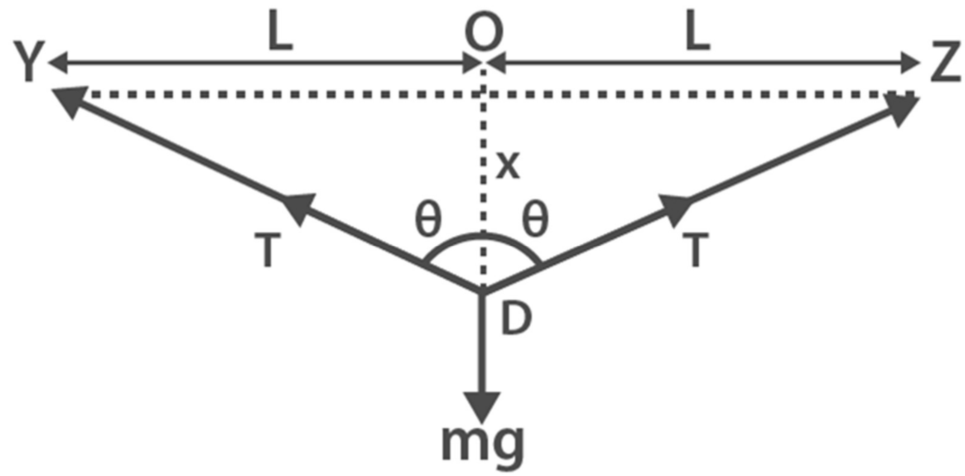
Q21. The Marina trench is located in the Pacific Ocean, and in one place, it is nearly eleven km beneath the surface of the water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume $0.32\ m^3$ is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches the bottom?

Answer:
Water pressure at the bottom of the trench, $p=1.1\times10^8$ Pa
Initial volume of the steel ball, $V=0.32 m^3$
Bulk modulus of steel, $B=1.6\times10^{11}\ Nm^{-2}$
The ball falls at the bottom of the trench, which is nearly 11 km beneath the surface of the water.
The volume change of the ball after reaching the bottom of the trench is $\triangle V$.
Bulk modulus, $B=p/(\triangle V/V) = pV/\triangle V$

$\triangle V = pV/B$
$=(1.1\times10^8\times0.32)/(1.6\times10^{11})$
$= 0.352 \times10^8/1.6\times10^{11}$
$= 0.22 \times 10^{-3} m^3$
The change in volume of the ball on reaching the bottom of the trench is $0.22 \times 10^{-3} m^3$

Q22. A mild steel wire of cross-sectional area 0.60×10^{-2} cm² and length 2 m is stretched (not beyond its elastic limit) horizontally between two columns. If a 100g mass is hung at the midpoint of the wire, find the depression at the midpoint.

Answer.



Let YZ be the mild steel wire of length 2l = 2m and cross-sectional area A = 0.60×10^{-2} cm². Let the mass of m = 100 g = 0.1 kg be hung from the midpoint O, as shown in the figure. And let x be the depression at the midpoint, i.e., OD

From the figure,
ZO =YO = l = 1 m
M = 0.1 KG
$ZD = YD = (l^2 + x^2)^{1/2}$
Increase in length, $\Delta l = YD + DZ - ZY$
= 2YD – YZ (As DZ = YD)
$= 2(l^2 + x^2)^{1/2} - 2l$
$\Delta l = 2l(x^2/2l^2) = x^2 / l$
Therefore, longitudinal strain = $\Delta l / 2l = x^2/2l^2$ .... (i)
If T is the tension in the wires, then in equilibrium 2Tcosθ = 2mg
Or, T = mg / 2cos θ

$= [mg (l^2 + x^2)^{1/2}] / 2x = mgl / 2x$

Therefore, Stress $= T / A = mgl / 2Ax$ . . . . (ii)

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